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A BRIEF SURVEY OF THE BERTH ALLOCATION PROBLEM[1]

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Resumo

O comércio internacional é altamente dependente do transporte marítimo e, portanto, os portos foram forçados a investir em infraestrutura e logística. Na literatura, existem muitas variações sobre como modelar e resolver o Problema de Alocação de Berços (BAP - como alocar navios em berços em um determinado horizonte de planejamento). Neste contexto, o presente trabalho estuda diferentes abordagens para a BAP. Comparações detalhadas são realizadas considerando variações de espaço e tempo, objetivos diferentes para otimizar e diferentes métodos de solução. Tais comparações são organizados em tabelas explicativas.

Palavras-chave: Alocação de Berços, Programação Linear Inteira.

Abstract

International trading is highly dependent on maritime transport and, therefore, ports have been forced to invest in groundwork and logistics. In the literature, there are many variations on modeling and solving the Berth Allocation Problem (BAP - how to allocate ships to berths in a given planning horizon). In this context, this paper surveys different approaches to the BAP. Detailed comparisons are conducted regarding time and space variations, different objectives to optimize, and different solution methods. Such comparisons are organized in explanatory tables.

Keywords: Berth Allocation, Integer Linear Programming.

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[1]Todos os autores assumem a responsabilidade pelo conteúdo do artigo.

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1. Introduction

Seaborne shipping is one of the most fundamental pillars of the global economic system throughout history. Nowadays, the increasing demand for ship transport is a major problem faced by marine terminals around the world. The massive flow of ships and containers in ports has generated a demand for logistics in order minimize the vessels' waiting and service time, the amount of discarded ships, among others. In addition, high competition among seaports, especially among those geographically close, also occurs as a result of this demand growth.

Many ports around the world have been subjects of study. According to Imai *et al*. (2001), in Japan's ports charges have been higher than those in other major hubs over several years. Part of the increased cost is the result of overcapitalization of the port for the relatively small cargo volume. Cordeau *et al*. (2005) and Giallombardo *et al.* (2010) conducted a study requested by Medcenter Container Terminal, located in the port of Gioia Tauro, which is linked to nearly 50 spoke ports. This terminal is geared towards transshipment activities involving mother vessels and feeders operating in hub-and-spoke system.

In this context, great difficulty in managing vessels has emerged, making it interesting to analyze questions about the Berth Allocation Problem (BAP) concerning ships in ports: how to allocate vessels to berths so as to mitigate some of these problems. Decisions as to where and when vessels should dock, for example, should be taken over a planning horizon, considering constraints of time and space, such as ship length, arrival time, number of containers for loading or unloading, and location of cargo. The port's characteristics, the products, and the demands generate numerous problems with small differences.

The Berth Allocation Problem reflects the leading decisions relating to financial resources, since proper allocation of vessels along the quay provides the satisfaction of owners of vessels and increases the port's productivity. Therefore, the continuous flow of cargo through terminals requires precise planning; decision analysis and optimization techniques should be applied to improve various operations.

Many of the problems studied in the literature are related to ports that work with loading and unloading of containers. The scale and nature of the problems faced by these terminals often make achieving optimal decisions difficult. Most berth containers at ports are granted to operators of ships, which are in charge of handling the load and, thus, achieve higher productivity. This is justified if they trade a large volume of containers with a large number of vessels, and then it may not be possible to guarantee cost savings. The terminal should be prepared to overcome the challenge of handling vessels with very high capacity. Key factors provide efficiency for stacking and transportation of this large amount of cargo. Handling high productivity containers at low cost is essential to the operation of the terminal and berths and cranes are the most important resources for this purpose. Efficient programming can contribute to a successful operation, which improves customer satisfaction, increases load flow, and leads to higher revenue from the port.

Most often, planning concerning the wharf and the crane are conducted separately, which may cause conflicts and lead to an unworkable plan. Therefore, allocation of berths and cranes is a new trend in the field of operations research, organizing the positions of mooring, berthing time and cranes for vessels arriving. Simultaneous allocation can contribute to greatly reduce the terminal's operating costs.

2. Time and Space Variations of the BAP

Several models for the BAP have been proposed in the literature. The differences relate to some assumptions that are taken, for example, if the possibility of waiting for ships exists, if several ships can moor in the same berth, if the time of arrival of vessels is considered, if the service time is proportional to the size of the ship, among others. Based on problems that occurred in the port of Hong Kong, in Guam & Cheung (2004) it is considered a problem of allocation of berths for ships, allowing multiple vessels mooring per berth and considering vessels arrival times (dynamic problem) with the objective of minimizing the total weighted flow time (the sum of the waiting times and a ship's service - the weights reflect the relative importance of the vessels). The necessity of planning efficient berth allocation to manage containers emerged because berth space is very limited. A typical berth at container terminals can accommodate multiple vessels at the same time, and when no space is available the vessel needs to wait.

In most problems, the objective is to minimize the costs of the port and the ship. This involves costs of waiting, the late fees and the use of equipment such as cranes. In some variations of the problem other conditions hinder the objective function, such as priority service, value of cargo handled, and others may also be included. In several container terminals, the administration has striven to reduce costs over the efficient use of resources (labor, work, berths, cranes, equipment). Berths are the most important resource and efficient allocation result in improved customer satisfaction and increased revenue from the port. Thus, in Kim & Moon (2003), by using an analytical approach, a model was proposed that seeks to maximize the utilization of wharfs in order to satisfy some constraints of container vessels. This is attained by minimizing the cost of penalty resulting from delays in ship departures and additional service costs resulting from unfavorable vessel location. According to these researchers, the position of every vessel is an important decision variable: if a ship is positioned close to the storage of containers to be loaded, then the delivery cost for the containers can be decreased.

Some ports in Europe and China, where there is intense traffic of vessels and containers, are called multi-users terminals: terminals with a long quay, where a large number of incoming vessels are simultaneously and dynamically allocated, not always in the same position. This type of terminal, according to Imai *et al*. (2005), is a widespread system in use in busy container ports, since productivity depends on efficient allocation of vessels. Thus, the terminal is partitioned into several berths, and allocation of vessels to the proper location is based on the characteristics of these berths, such as length and depth. The associated problem is called Discrete Berth Allocation Problem - DBAP. However, the article is based on the fact that sometimes ships are moored across the border from berth boundary, or the mooring of vessels is performed on a continuous space (Continuous Berth Allocation Problem - CBAP).

In Bierwirth & Meisel (2010), the problem is classified into a few categories. The vessel arrival process can be considered static or dynamic. In the first case, arrival time imposes no restriction on timing for mooring: vessels can berth at any time given that a portion of the quay is available. In the second case, vessels are not allowed to berth before the expected arrival time. Regarding the arrival of vessels, it may be deterministic, when the expected arrival times are known, or stochastic, in which the distribution of arrivals is dependent on uncertainties concerning the schedules. In Cordeau *et al*. (2005) a model for the discrete version of the problem is introduced. The formulation is capable of handling a weighted sum of service times as well as windows on berthing times. Table 1 summarizes the classification about the berth allocation and arrival process for each of the papers cited by this survey.

Service times can also have different classifications: fixed and unchangeable, according to the position to where the vessel was allocated, or stochastic (interruptions can happen, bringing uncertainty in service times). Most works found in literature allow no interruptions. Regarding space restrictions, in addition to the two proposed by Imai *et al*. (2005), in Bierwirth & Meisel (2010) the hybrid case is considered, in which the quay is partitioned into berths but one large vessel can occupy more than one berth and small vessels may share a berth.

Table 1: Different approaches to the BAP.

	Berth Allocation			Arrival	
Article	**Discrete**	**Continuous**	**Hybrid**	**Static**	**Dynamic**
Bierwirth & Meisel (2010)			Survey		
Buhrkal *et al*. (2011)	X				X
Chen *et al*. (2007)			Equipment scheduling: BAP was already solved		
Cordeau *et al*. (2005)	X				X
Dai *et al*. (2008)		X		X	
Frojan *et al*. (2015)		X			
Giallombardo *et al*. (2008)	X				X
Giallombardo *et al*. (2010)	X				X
Guam & Cheung (2004)	X			X	
Hansen & Oğuz (2003)	X			X	X
Imai *et al*. (2001)	X				X
Imai *et al*. (2005)		X			X
Imai *et al*. (2008)	X			X	
Kim & Moon (2003)		X		X	
Lalla-Ruiz *et al*. (2014)	X				X
Liang *et al*. (2009)	X			X	
Liu *et al*. (2006)			Equipment scheduling: BAP was already solved		
Lopes *et al*. (2011)	X				X
Nishimura *et al*. (2001)	X			X	
Nishimura & Papadimitriou (2001)	X			X	
Oliveira *et al*. (2010)	X				X
Pomari & Chaves (2014)	X				X
Silva *et al*. (2008)	X				X
Stahlbock & Vob (2008)			Survey		
Theofanis *et al*. (2007)	X				X
Türkoğullari *et al*. (2014)		X			X
Umang *et al*. (2012)			X		X
Vacca *et al*. (2013)	X				X
Yang & Wang (2010)	X				X
Zhang (2010)			Equipment scheduling: BAP was already solved		

There is also another way to divide the physical space of the terminal. In the problem studied by Dai *et al*. (2008), the terminal is divided into several wharfs. Each one consists of a set of multiple berths, which in turn are divided into sections. Each section corresponds to a linear stretch of space. Each vessel can be physically moored through different sections, but cannot be moored through different wharfs. In this article, the goal is to design an efficient berth planning system to allocate berth space to vessels in each planning horizon, ensuring that all vessels are moored as they arrive at the terminal and are near the locations of preference. Such allocation shall be determined dynamically over time. Frojan *et al*. (2015) propose a continuous BAP with multiple quays. The problem consists of deciding the berthing time, the quay, and the position at the quay for each vessel, in such a way that the total assignment cost is minimized.

In addition to these aspects, a formulation with further extensions is proposed in Hansen & Oğuz (2003). It considered safety constraints due to a ship’s length or draft, due dates for the departures of some or all ships, and a bounded planning horizon.

In the literature, a few variations of the BAP can be found. In Giallombardo *et al*. (2008), one variation is called Tactical Berth Allocation Problem (TBAP). The difference between this problem and the BAP is the length of the planning horizon: planning in TBAP consists in months, enabling integration among terminal costs in a more comprehensive way. In a transshipment terminal, containers arrive and depart by vessels while being temporarily stored in the yard. When a vessel is unloaded, containers should be allocated to yard positions close enough to the vessel berthing point, accelerating the service of that vessel. However, when a container’s departure position is far from its yard position, such container must be repositioned before the arrival of the vessel. Consequently, the yard management deals with a dynamic positioning of containers through their duration-of-stay inside the terminal. It is worth noting that tactical berth allocation determines the long-term favorite berth (home berth) for each vessel, thus inducing container flows inside the yard. Thus, yard costs are an effect of the simultaneous assignment of vessels to home berths. Another difference concerns the vessels’ arrival times. In the TBAP, shipping lines indicate time ranges for expected arrival times. Tactical berth plan must accommodate for such arrival times or an alternative agreement should be sought. Therefore, the issue is to determine if accommodating a customer request is feasible and how it impacts the whole terminal operations such as yard costs and quay crane utilization. Furthermore, since handling times are influenced by many factors, such as the amount of loading and unloading containers, the distribution of these containers inside the vessels, the number of quay cranes assigned to each work shift, the TBAP deals with the negotiation between the terminal and the shipping lines as to the reserved assignment of quay cranes along work shifts. The TBAP is also explored in

Giallombardo *et al*. (2010). The berth allocation problem is integrated with the quay crane assignment problem in a level two decision problem. Vacca *et al.* (2013) considered the same problem defined. The joint optimization problem aims at assigning a berthing position and a quay crane assignment profile to every vessel over a given time horizon as well as at scheduling incoming vessels according to their time windows.

Stahlbock & Vob (2008) discuss that container terminal systems might have constituent parts such as handling equipment, human resources, and assisting systems. Cranes are fundamental equipment in the terminal used for the handling of containers. Inefficient use of these may be the bottleneck for fast operations on vessels. As a result, the problem of crane allocation (Quay Crane Allocation Problem - QCAP) arises: after a vessel arrives at the terminal, cranes should unload containers from it, which are taken to the stockyards. The same applies to a vessel to be loaded for export containers. In QCAP, it is assumed that a vessel is already positioned in a berth and waiting to be serviced by cranes. It is very expensive to have a vessel remain waiting in the mooring position. Thus, it is important that as many cranes are used as needed, maximizing the number of containers loaded or unloaded per time unit. However, these devices are expensive and certainly not a cost-effective investment. Consequently, a competitive terminal must maximize the efficiency of operations on a limited number of cranes.

According to Zhang (2010), there are two rules that restrict the operation of cranes. The first is the restriction of non-crossing: cranes should be kept in a fixed sequence, as long they share a common rail along the berth. The second restriction is a minimum separation: cranes need to maintain a minimum distance from one another for safety reasons. Here, the problem of dynamic crane allocation is studied: a fixed number of quay cranes at a berth with limited length serve a series of vessels that arrive sequentially at the berth over time. The goal is to maximize the long-run average throughput (number of containers handled per time unit) subject to the non-crossing and minimum separation restrictions. This problem presents complications resulting from weather conditions, containers lost, and traffic congestion at the terminal. For this reason, most studies found in the literature solve the problem in a static configuration, in which a given number of vessels at a berth are pre-assigned to a set of quay cranes during a planning horizon. The problem is formulated as a machine scheduling problem: if any parameter changes in the system, then the problem needs to be solved again. However, the static case is not optimal and tedious to implement over an extended period of time.

Table 2: Different objectives to optimize.

Article	Objective Function
Bierwirth & Meisel (2010)	Survey
Buhrkal *et al*. (2011)	Minimize the total waiting and handling times of every ship
Chen *et al*. (2007)	Minimize the makespan (the time it takes to serve a given set of ships)
Cordeau *et al*. (2005)	Minimize the sum for each ship of the service time
Dai *et al*. (2008)	Minimize the makespan (the time it takes to serve a given set of ships)
Frojan *et al*. (2015)	Minimize total assignment cost
Giallombardo *et al*. (2008)	Maximize the total value of chosen QC profiles and minimize the housekeeping costs generated by transshipment flows between ships
Giallombardo *et al*. (2010)	Maximize the total value of chosen quay crane profiles and minimize the housekeeping costs generated by transshipment flows between ships
Guam & Cheung (2004)	Minimize the total weighted flow time
Hansen & Oğuz (2003)	Minimize the total completion time (waiting and handling time for all ships)
Imai *et al*. (2001)	Minimize the total waiting and handling times for every ship
Imai *et al*. (2005)	Minimize the sum of waiting and handling times for every ship
Imai *et al*. (2008)	Minimize the total service time
Kim & Moon (2003)	Minimize the penalty cost resulting from delay and the additional handling costs resulting from non-optimal locations
Lalla-Ruiz *et al*. (2014)	Maximize the sum of the values of the chosen quay crane profiles and minimize the yard-related housekeeping cost
Liang *et al*. (2009)	Minimize the sum of the handling time, waiting time, and delay time for every ship
Liu *et al*. (2006)	Minimize the maximum relative delay of vessel departures
Lopes *et al*. (2011)	Minimize the service time of each vessel at the wharf
Nishimura *et al*. (2001)	Minimize the total service time incurred by ships
Nishimura & Papadimitriou (2001)	Minimize the sum of waiting and handling times for every ship
Oliveira *et al.* (2010)	Minimize the sum of all service times
Pomari & Chaves (2014)	Minimize the sum of length of service, linked to an operating cost
Silva *et al*. (2008)	Minimize the total allocation cost
Stahlbock & Vob (2008)	Survey
Theofanis *et al*. (2007)	Minimize the total weighted service time of all the vessels.
Türkoğullari *et al*. (2014)	Minimize the total cost: deviation from the desired berth section, berthing later than the arrival time and departing later than the due time
Umang *et al.* (2012)	Minimize the total service time
Vacca *et al*. (2013)	Maximize the difference between the revenue associated with the chosen quay crane profiles and the housekeeping costs generated
Yang &Wang (2010)	Minimize the turnaround time of the arriving ships and minimize the transportation and handling cost.
Zhang (2010)	Maximize the long-run average throughput

A large vessel can be simultaneously processed by multiple cranes. Typically, the handling time of a vessel decreases when more cranes are assigned to it. Similarly, the handling time of a vessel is greater than expected if an insufficient number of cranes are operating, delaying the vessel's departure. Unfortunately, it is not possible to assign many cranes to the same vessel: the equipment is expensive and the terminals do not have many of them, which makes an opportunity cost in a real loss if there is an idle crane. In addition, many cranes in one vessel may cause interference rather than improvement. In Liu *et al*. (2006), some assumptions are made in order to simplify the complexity of the modeled problem. Firstly, the QCAP always takes the result of the BAP: it is considered that the berth allocation problem has been solved and berthing positions, arrival and departure times, and vessels' workloads are input data for the crane allocation problem. Then, each vessel is divided longitudinally into bays; each bay accommodates a row of container stacks; bays in all vessels are of the same length. Thus, the lengths of vessels are represented in (total) number of bays (bay lengths). The total length of the berth is also represented in number of bays. Hence the cranes are considered identical, both in terms of productivity in loading/unloading containers and in terms of speed of moving from bay to bay. The safety distance between adjacent QCs is also represented in number of bays. This safety distance is nonzero and, therefore, only one QC can work on a bay at a time. Once a QC starts processing a bay, it leaves only when it has finished this bay's workload. The total preparation time on the quayside for a docking vessel to depart and, if applicable, for a waiting vessel to dock, is assumed to be a constant for all vessels. So the goal is to minimize the vessel processing time.

In general, BAP and QCAP occur in conjunction, resulting in a simultaneous allocation problem (B&CAP), as shown in Imai *et al*. (2008). B&CAP consists of two subproblems: BAP and QCAP. For the first subproblem, the same DBAP model used in Imai *et al*. (2001) is considered. To constitute B&CAP, the following constraints are added to QCAP: cranes get through an idle berth having some cranes present by the pushing-in and pulling-out procedure and cannot move from one berth to another via other berths if the other berths are engaged in ship handling. In this case, the movement of the cranes is treated in more details, and for this new decision variables must be defined. The objective is to minimize the total service time. In Türkoğullari *et al.* (2014), the integrated planning of berth allocation and quay crane assignment (BACAP) is formulated as a binary integer linear program and then it is extended by incorporating the quay crane scheduling problem (BACASP).

The objectives in previous studies of the simultaneous problem are almost always the same: minimize the period of stay (or delay) of the vessels. However, in Yang & Wang (2010), it is noteworthy that, despite the possibility of increasing the satisfaction of ship companies and the competitiveness between ports, it can have negative effects, such as high cost of production. Effective planning should consider both. Taking into account this consideration, a model that minimizes the average time of vessel return and the production costs at the same time is established.

Table 2 lists each of the objective functions that were already studied in some paper.

3. Different Ways of Solving the BAP

Each of the models above is favored by a different method of resolution. Genetic Algorithms are often used because of their potential as a singular optimization technique.

Theofanis *et al*. (2007) proposed a GA based heuristic algorithm to solve the Discrete and Dynamic BAP. An integer chromosomal representation was used in order to fully exploit the characteristics of the problem. The crossover was not allowed because it would lead to a large number of infeasible solutions that would either be discarded or mutated to produce feasible solutions. Instead, at each generation, insert, swap, inversion, and scramble mutation were applied to all chromosomes. Nishimura *et al*. (2001) applied GA to the berth allocation problem with the multi-ship service environment and two different types of chromosome representations were employed.

In Silva *et al*. (2008), a simplified genetic algorithm (GA) was proposed which can analyze a list of vessels (arrival time, draft size[1], length and type of cargo) and a list of berths (with schedules, availability, depth and length) in order to determine if the restrictions are satisfied and subsequently allocate a berth to a vessel. The algorithm has as a criterion for allocation check at the last moment to release a given berth, add the interval period to be considered and allocate this berth to a vessel. Thus, the vessel sequence is represented by a chromosome and an initial population is constructed: individuals are encoded in a sequence of vessels, each component and the sequence is a gene (vessel). The performance evaluation represents the individual's ability to adapt to the environment, and this measure should be considered so that the lower its value, the greater its ability to adapt. At first the individuals are sorted in descending order of performance (the first is the best performance). The crossover is conducted by considering the average of the positions taken by the individuals in

[1] Depth that is the lowest point of a ship.

the chromosomes. Then they are sorted in a list in ascending order and mutation is performed using an occurrence probability.

In Liang *et al*. (2009), a hybrid approach is suggested for QCAP that combines GA with a heuristic. It is known that pure GAs are able to locate the promising regions for global optimum in a search space, yet sometimes they have difficulty in finding the exact minimal solution. The motivation behind hybridization concept is usually to obtain better performing systems that harness and unite advantages of the individual pure strategies. Since it is likely that the solutions found by GAs can still be improved by using other solution methods, the proposed hybridization is likely to facilitate local convergence. The procedure can be divided into four phases. In the first phase, a sequence of vessels is created: the precedence relationship of vessels is determined, a random priority is generated for each sequence and it is determined by the First-Come-First-Served (FCFS) rule. In the second, the vessels are allocated to berths by generating a random number for each vessel. In the third, the cranes are assigned to berths by generating a random number for each berth. In the fourth phase, the berth assignment and crane scheduling occur: quay cranes are transferred among berths satisfying the quay cranes assignment and a Draw Gantt chart for scheduling is made. Thus, the procedure of Cross-Over is achieved by a one-cut point method, which randomly selects one cut point. To ensure the feasibility of the new offspring, it is combined with a mapping strategy. For the procedure of mutation, the swap mutation was adapted to generate the new offspring.

A variation of the GA was also used in Yang & Wang (2010) to solve the problem of berth and crane allocation. A solution method of three phases was suggested. The first phase consists of a multi-objective genetic algorithm (MOGA) to generate or iterate feasible solutions which may define the order of service, berthing positions and the number of cranes for each ship. The second phase was to gain the exact berthing time by a quay crane scheduling heuristic. The last one was to select a final schedule from the Pareto solutions (a solution is Pareto optimal if it is not possible to improve the situation without worsening the situation of another agent).

The problem of simultaneous allocation B&CAP in Imai *et al*. (2008) was solved using a heuristic based on GA to estimate an approximate solution. At each iteration (generation), the entire procedure performs two computations: one is the GA procedure (crossover and mutation) for ship-berth-order allocation without consideration of crane scheduling and the other is the heuristic for scheduling cranes given a ship-berth-order allocation alternative defined by a chromosome. The genetic algorithm is characterized by transformation of the objective function value into a fitness value which is evaluated for reproduction. Because a chromosome simply defines only a ship-berth-order assignment without any information on the crane schedule. Consequently, before the reproduction process, crane scheduling is performed given a chromosome, so the objective function value is produced to be utilized for the fitness value. During reproduction, there is a process in which individual chromosomes are copied according to their scaled fitness function value. The tournament strategy is provided. For the crossover procedure, the so-called 2-point crossover is employed. For the phase of crane scheduling, the feasibility of ship service schedule is determined as to the number of available cranes and crane tasks are rescheduled. The chromosome determines the ship-berth-order assignment. If there are many cranes, ships are served as soon as they arrive at the port and assigned berths become idle, according to the service order of ships at the respective berths, which is scheduled by a chromosome generated (such ship service schedule is hereafter referred to as initial schedule). However, if there are not enough cranes, some ships may delay their services even with a well-organized crane schedule. It is therefore necessary to examine whether the number of cranes given to the system as an input parameter is large enough for ships to be served in a starting lineup. Thus, service delay is minimized by optimum scheduling of crane tasks.

The berth allocation model can be represented in a space-time diagram in which the horizontal axis represents the planning horizon and the vertical axis represents the berth length. Ergo, the ship is interpreted as a rectangle whose length is the processing time and whose height is the ship size. This model has been formulated in two ways in Guam & Cheung (2004). The first is called the Relative Position Formulation (RPF), which considers the relative position of the rectangles in the diagram. For this case, there is an optimal solution in which each vessel rectangle j is immediately right of another vessel rectangle or starting at period aj, and is immediately above another vessel rectangle or starts at berth section 1. The other is called the Position Assignment Formulation (PAF), which the area covered by the rectangles is considered. The diagram is partitioned into blocks, each having a height of one berth section and the width of one time unit. The Lagrangian Relaxation based procedure (LR) is proposed, which provides lower bound to the SRP. For best bounds, the classical subgradient method should be used. Then, a tree search procedure is described which uses the properties of both formulations, in which the rectangles are added to the space-time diagram one by one. The search tree is branched as follows. The k inserted vessel rectangles form a stair on each tree node. The steps of the stair help indicate potential feasible positions for inserting other vessel rectangles. When we insert a vessel rectangle on one step, the assignment time of the vessel rectangle is the maximum of the step start time and the vessel arrival time. To reduce the branching, upper and lower bounds are used to generate simplification of the process. In this paper, a heuristic is also proposed which groups the vessels in batches according to their arrival times, generating an initial solution. Then the ships are exchanged in two consecutive batches if the permutation reduces the objective function value. Only then the exact search tree procedure is applied to each resulting batch.

This representation is also used in Imai *et al*. (2005), but with the axes reversed (rectangle length represents ship length and rectangle height represents service time). The bottom edge of the rectangle represents the ship handling start time. The top edge represents the ship handling completion time. Quay space can be geometrically represented by infinitely long rectangles, whose lengths represent quay length and whose height defines the time. When two vessels are positioned side by side, some distance off must exist between them, which is added to ship length. Moreover, some time off is also observed when a ship is positioned in a berth where a vessel has been previously served, which will be added to each vessel's service time. The continuous BAP (CBAP) studied here is to determine the coordinates of the rectangles along the quay so all vessel rectangles are neither overlapped nor allocated below their predetermined horizontal lines representing the vessels' arrival time. The upper bound for the BAPC is obtained from the optimal solution of the discrete BAP (DBAP) when the berth length is determined by the maximum length of the ship involved in the problem. The lower bound is obtained when the berth length is determined by a very small value.

In Kim & Moon (2003), BAP solutions are encoded into a sequence of vessels to apply the Simulated Annealing algorithm (SA) to the berth-scheduling problem. To decode a sequence of vessels into a solution, it must be possible to obtain near-optimal positions of vessel rectangles in the time-wharf plane from the given sequence of vessels. A x-cluster is defined as a set of rectangles (vessels) whose vertical sides are in contact with one another. Also, a y-cluster is defined as a set of rectangles (vessels) whose horizontal sides are in contact with one another. A cluster of vessels is stable if the total cost for vessels in the cluster cannot be reduced by moving the cluster in the positive or negative direction. When an x-cluster is stable, either one or more rectangles in the x-cluster are located at their least-cost points on the x-axis or a side of at least one rectangle in the cluster is on the right or the left boundary of the wharf. When a y-cluster is stable, either the berthing times of all vessels in the y-cluster are located in the range of [arrival time, operating time - departure time] or the berthing time of one or more vessels in the cluster is at its arrival time or zero.

Following the same interpretation, in Dai *et al*. (2008) berthing plans are encoded by pairs of permutations (H,V) of all vessels. A directed graph is associated with the constraints involving the decision variables of time and a time-cost problem is formulated. Another graph is associated with the constraints involving the decision variables of space and then a space-cost problem is formulated. The approach suggests an effective method to obtain a good pair of packing given a fixed sequence that is sought in the space with an algorithm of Simulated Annealing.

The Clustering Search (CS) presented in Oliveira *et al*. (2010) is mainly composed of three main components: a metaheuristic that generates solutions, a grouping process, and a local search heuristic. At each iteration, a solution S is generated by the metaheuristic and sent to the grouping process. Then, it is grouped in the same cluster Cj and the center of this cluster ci is updated with information from new grouped solution, leading to a displacement of center in the search space. Then the cluster volume vj is analyzed and if it reaches a limit value λ it means that some standard solution is predominantly generated and this cluster may be in a promising region of search. Finally, the inefficiency index rj is analyzed: if the local search heuristic does not improve the solution, a random perturbation is applied in the center to avoid a possible local optimum. Otherwise, the local search heuristic is applied to the center analyzing the cluster neighborhood. Once this process is over, the metaheuristic is taken again to generate a new solution.

In Buhrkal *et al*. (2011) and Umang *et al*. (2012), the BAP is formulated as a Generalized Set Partitioning Problem (GSSP). Assuming that all measurements of time are considered integer, a column (variable) represents a feasible assignment of a single ship to a berth at a specific time. The first n rows correspond to the n vessels. If a column represents an assignment of ship i, there will be a 1 in row i and zeros in the rest of the n first rows. Furthermore, there is one row for each available time unit at each berth. An entry in a column is equal to one if the vessel occupies the berth at the considered time unit, otherwise it is zero. The cost of each column is equal to the service time arising from the ship/berth/time assignment.

The mixed integer model that was proposed in Chen *et al*. (2007) is an extension of the classical flow shop with one machine at each stage. The objective is to minimize the makespan to serve a set of loading and unloading ships in a given time horizon improving the connection between the equipment and increasing production. Thus, it was solved by an algorithm based on tabu search. This method enables moving away from the current solution that makes the objective function worse in the hope that a better solution is achieved. Based on this, a completely new neighborhood structure is proposed to deal with the problem of scheduling with blocking constraint. In Giallombardo *et al*. (2010), tabu search is combined with mathematical programming techniques to solve the TBAP. Tabu search was also developed for the discrete case in Cordeau *et al*. (2005). It was then extended for the continuous case. Lalla-Ruiz *et al*. (2014) proposed a biased random key genetic algorithm (BRKGA) to solve this problem so as to obtain good quality solutions with considerably short computational effort. BRKGA was also used in Pomari & Chaves (2014) to solve the discrete and dynamic BAP.

Another approach in Lopes *et al*. (2011) is based on application of the Greedy Randomized Adaptive Search Procedure (GRASP) to build solutions integrated with the Path Relinking (PR) method as a search intensification strategy to solve the BAP. Each GRASP iteration consists of a constructive phase, in which a feasible solution is constructed, and a local search phase, which applies iterative motion until the optimal location is found based on the constructed solution. Then, PR is applied as an intensification strategy exploring trajectories that connect the solution of the current iteration to the best solution obtained so far.

Table 3 compares the method used to solve the different variations of the BAP proposed by each of the articles analyzed in this paper.

4. Conclusions

This article covered some of the several variations of the Berth Allocation Problem. Some of them concern the modeling of the problem: different physical, temporal, spatial and even commercial characteristics are found in the literature (Table 1). There are also different interesting objectives to be optimized (Table 2). In addition, each model benefits from a specific solution method, as shown before. Table 3 shows that there are many approaches to solve each model properly, such as genetic algorithm, local search, hybrid and exact methods and others.

It is noteworthy that it is a very comprehensive subject, and the difficulty and complexity vary widely.

Table 3: Different ways to solve the BAP.

Article	Method
Bierwirth & Meisel (2010)	Survey
Buhrkal *et al*. (2011)	GSSP: Time is discretized and for each berth and at most one ship can be at the berth at any given time.
Chen *et al*. (2007)	Tabu search: a new neighborhood structure is proposed for TS to deal with the problem of scheduling with blocking constraint.
Cordeau *et al*. (2005)	Tabu search heuristic.
Dai *et al*. (2008)	Local search: The neighborhood structure is defined by use of the concept of sequence pair.
Frojan *et al*. (2015)	GA
Giallombardo *et al*. (2008)	The formulations were tested with CPLEX 10.2.
Giallombardo *et al*. (2010)	It combines tabu search and mathematical programming techniques.
Guam & Cheung (2004)	The tree search procedure and pair-wise exchange heuristic are combined.
Hansen & Oğuz (2003)	The formulations were tested with CPLEX.
Imai *et al*. (2001)	Heuristic procedure based on the Lagrangian relaxation
Imai *et al*. (2005)	Heuristic: The problem is solved in two stages: identify a solution given the number of partitioned berths and reallocate the ships.
Imai *et al*. (2008)	GA: the procedure of determination of berth scheduling and crane scheduling are iterated at the same time.
Kim & Moon (2003)	SA: To obtain near-optimal positions of vessel rectangles in the time-wharf plane from the given sequence of vessels.
Lalla-Ruiz *et al*. (2014)	BRKGA
Liang *et al*. (2009)	Hybrid approach: It combines GA with heuristic to find an approximate solution for the problem.
Liu *et al*. (2006)	Equipment scheduling: BAP was already solved
Lopes *et al*. (2011)	It applies GRASP to build solutions and implements PR as a search intensification strategy.
Nishimura *et al*. (2001)	Heuristic procedure based on the GA.
Nishimura & Papadimitriou (2001)	A solution procedure is used which employs the subgradient method based on the Lagrangian relaxation.
Oliveira *et al*. (2010)	Hybrid approach: CS & SA as solutions generator.
Pomari & Chaves (2014)	BRKGA as solutions generator to the CS´s clustering process.
Silva *et al.* (2008)	GA: a chromosome is represented by a sequence of vessels.
Stahlbock & Vob (2008)	Survey
Theofanis *et al.* (2007)	GA based heuristic algorithm
Türkoğullari *et al*. (2014)	Propose a necessary and sufficient condition for obtaining an optimal solution and a cutting plane algorithm.
Umang *et al*. (2012)	Heuristic approach based on the principle of squeaky wheel optimization.
Vacca *et al*. (2013)	Column generation and Branch and Price Algorithm
Yang &Wang (2010)	Three-phase solution method: MOGA to generate feasible solutions, quay crane scheduling heuristic, and select a final schedule from the Pareto solutions.
Zhang (2010)	Equipment scheduling: BAP was already solved

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